day 12

2.6 Continuity - the first important use of limits

IMPORTANT

A function $f(x)$ is continuous at $x=a$ iff

i) $f(x)$ is defined at $x=a$

ii) $\lim_{x\to a} f(x)$ exists

iii) $\lim_{x\to a} f(x) = f(a)$

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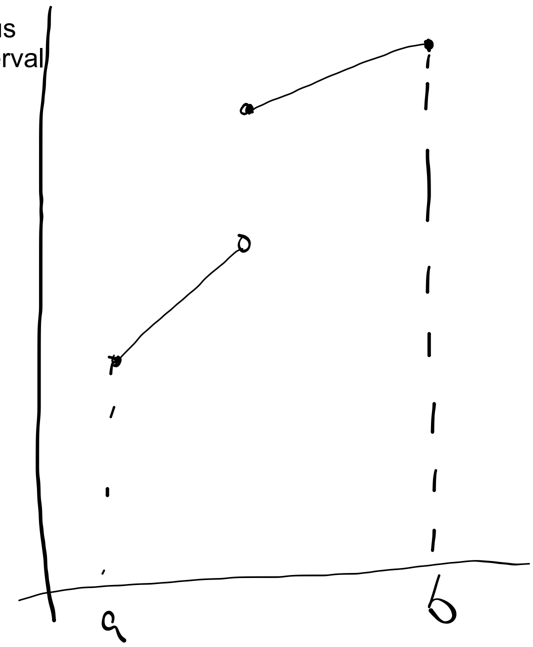
IVT — broken!

If f(x) is continuous on the closed interval from a to b, then

For every L in the open interval from f(a) to f(b)

There exists a c with f(c)=L



IVT

If $f(x)$ cont on $[a, b]$, then

$\forall L \in (f(a), f(b))$

$\exists c$ with $f(c) = L$

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IVT —— broken!

If f(x) is continuous on the closed interval from a to b, then

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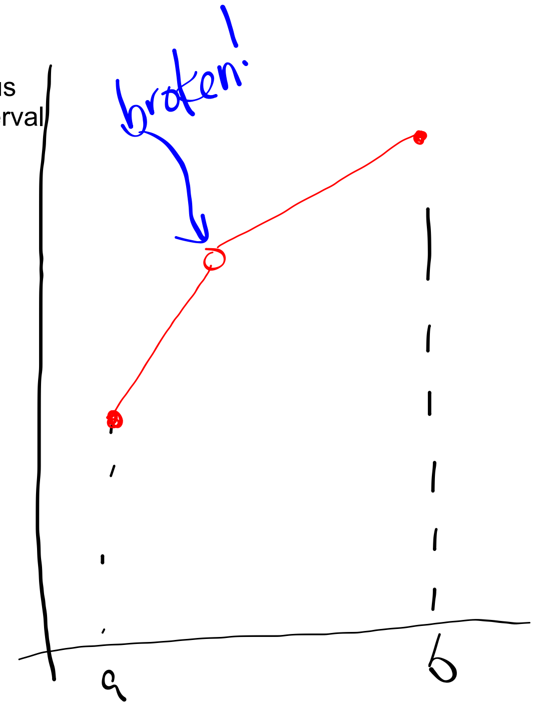

IVT

If $f(x)$ cont on $[a,b]$, then

$\forall L \in (f(a),f(b))$

$\exists c$ with $f(c)=L$

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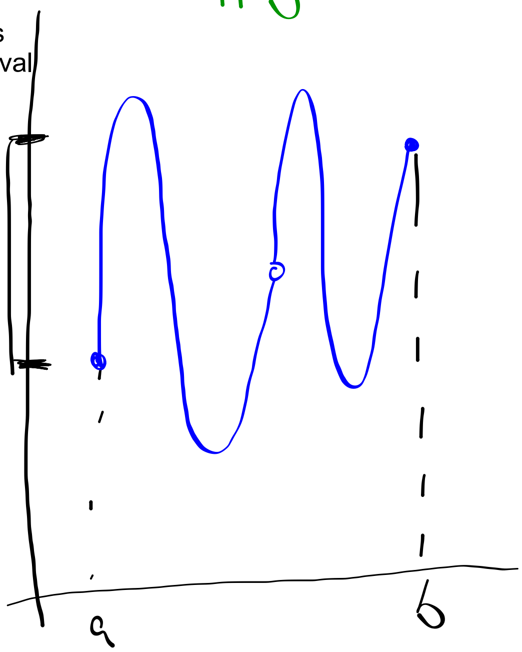
IVT —— broken!

not "broken" it just doesn't apply

If f(x) is continuous on the closed interval from a to b, then

For every L in the open interval from f(a) to f(b)

There exists a c with f(c)=L



IVT

if f(x) cont on $[a,b]$, then

$\forall L \in (f(a), f(b))$

$\exists c$ with $f(c)=L$

IVT applies:

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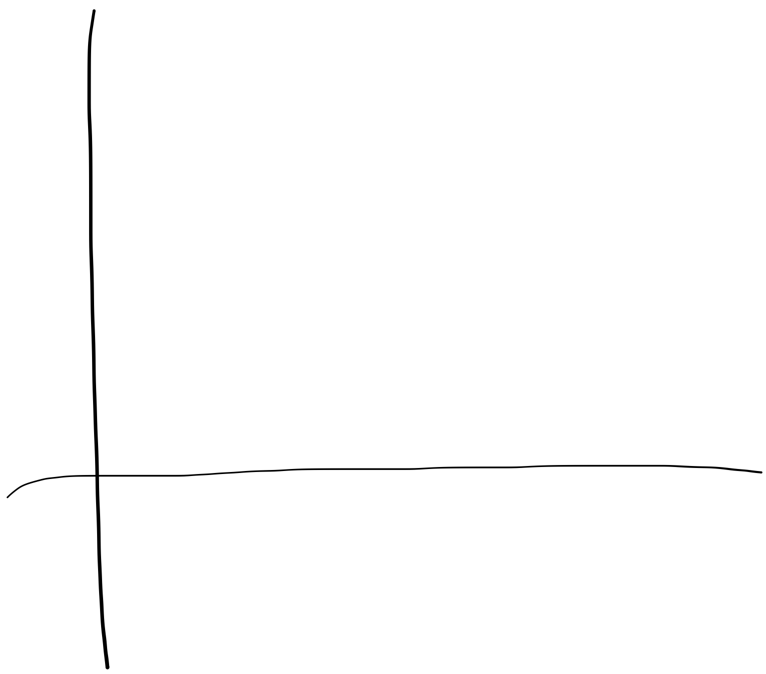

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2.3 / 77-78, 83

2.4 / 35-38

2.6 / 1-6, 9-14

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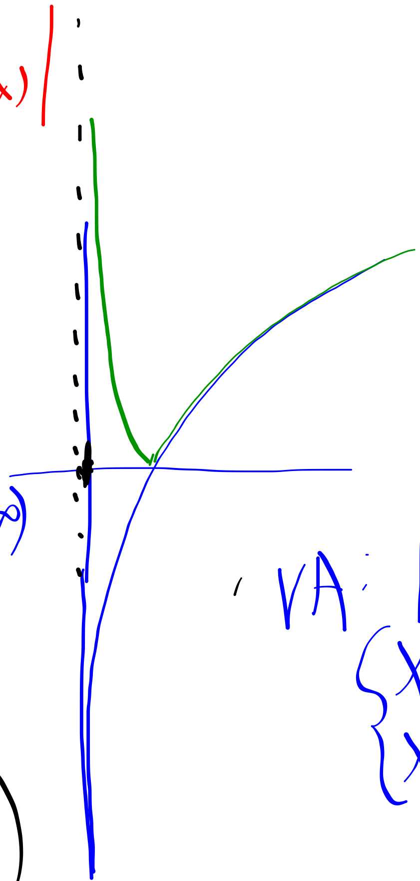
2.5)
42 $f(x) = |\ln(x)|$

$\lim_{x\to-\infty} |\ln x| = DNE$

$\ln x$



$|\ln(x)|$
range $[0,\infty)$

$\ln(x)$
Domain: $(0,\infty)$
Range: $(-\infty,\infty)$

$\lim_{x\to\infty} |\ln(x)| = +\infty$

VA: $\lim_{\left\{\begin{array}{l} x\to a^+ \\ x\to a^- \end{array}\right.} f(x) = \left\{\begin{array}{l} +\infty \\ -\infty \end{array}\right.$

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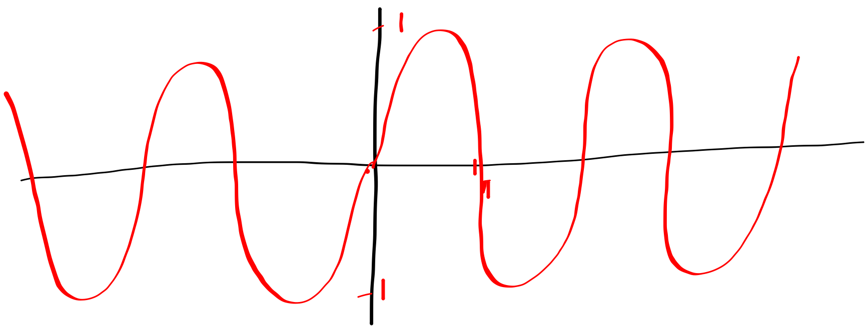
43) $f(x) = \sin(x)$



the end behavior is bounded $[-1, 1]$

end behavior

$\lim_{x \to \infty} \sin x = DNE$ $\lim_{x \to -\infty} \sin x = DNE$

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35) $f(x) = \dfrac{4x^3+1}{2x^3+\sqrt{16x^6+1}}$

$$= \frac{x^3\left(4+\frac{1}{x^3}\right)}{x^3\left(2+\frac{1}{x^3}\sqrt{16x^6+1}\right)} \qquad x \neq 0$$

$$= \frac{x^3\left(4+\frac{1}{x^3}\right)}{x^3\left(2+\frac{1}{x^3}\sqrt{x^6}\sqrt{16+\frac{1}{x^6}}\right)} \qquad x \neq 0$$

$$\lim_{x\to\infty} \frac{x^3\left(4+\frac{1}{x^3}\right)}{x^3\left(2+\underbrace{\frac{1}{x^3}\,x^3}_{1}\sqrt{16+\frac{1}{x^6}}\right)} = \lim_{x\to\infty} \frac{x^3\left(4+\frac{1}{x^3}\right)}{x^3\left(2+\sqrt{16+\frac{1}{x^6}}\right)}$$

$\frac{1}{x^3} \to 0$, $\frac{1}{x^6} \to 0$

$$= \frac{4}{2+\sqrt{16}} = \frac{4}{6} = \frac{2}{3}$$

but

$$\lim_{x\to-\infty} \frac{x^3\left(4+\frac{1}{x^3}\right)}{x^3\left(2+\frac{1}{x^3}\sqrt{x^6}\sqrt{16+\frac{1}{x^6}}\right)}$$

$\sqrt{x^6} = -x^3$

$\boxed{(-2)^6 = 64 \quad \sqrt{64} = 8 \quad (-2)^3 = -8}$

$$= \lim_{x\to-\infty} \frac{\left(4+\frac{1}{x^3}\right)}{2+\frac{1}{x^3}(-x^3)\sqrt{16+\frac{1}{x^6}}} = \frac{4}{2-\sqrt{16}} = \frac{4}{2-4} = -2$$